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The Prediction of Evaporation, Drainage, and Soil Water Storage for a Bare Soil[1]

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ABSTRACT

Evaporation, drainage, and changes in storage for a bare Plainfield sand were measured with a lysimeter during June, July, and August 1967, under natural rainfall conditions. Cumulative evaporation at any stage was proportional to the square root of time following each heavy rainfall. The drainage rate was found to be an exponential function of water storage. Both relations can be predicted from flow theory with knowledge of soil capillary conductivity, diffusivity, and moisture retention characteristics. Using these two relations and daily rainfall data, the water storage in the top 150 cm was predicted over the season to within 0.3 cm.

Additional Key Words for Indexing: lysimeter.

A SATISFACTORY knowledge of the soil profile water storage is important to irrigation management and in many hydrological problems. Infiltration, evaporation, and deep percolation depend, in some measure, upon the water content of the soil profile. A number of laboratory studies such as those by Gardner and Hillel (1962), Gardner and Gardner (1969), Youngs (1960), and Gardner (1962) suggest that it may be possible to describe the relation of evaporation and drainage to soil water content by relatively simple though somewhat approximate expressions based upon the flow equation.

In this paper a simple solution of the flow equation for the drying of a semi-infinite soil profile is used to predict the evaporation from a sandy soil for a 3-month period during the summer. A relation between drainage rate and profile storage is obtained from lysimeter measurements and compared with that which would be predicted from flow theory. Since runoff was negligible during the experiment, these relations combined with rainfall data permit calculation of changes in profile storage.

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EXPERIMENTAL PROCEDURE

Lysimeter Description

Daily measurements of evaporation from the bare soil were obtained from a 35-metric ton, hydraulic load-cell lysimeter installed in the sand plains area of Wisconsin at the University of Wisconsin Hancock Experimental Farm (Black et al., 1968).

The dimensions of the inner tank are 5.5 m by 2.1 m in area by 1.5 m deep. The soil is a Plainfield sand. A tension drainage system with a network of porous stainless steel filter candles at the bottom of the lysimeter removes drainage at a suction of 30 cm of water, and the drainage is recorded. The resolution of the drainage system is, on a daily basis, 0.02 mm of water, while the resolution of the weighing system is 0.08 mm of water. Weighing system sensitivity and drainage calibration remained unchanged during the 3 months.

The profile of a cultivated Plainfield sand has a 25-cm plow layer having a small quantity of silt and organic matter, beneath which is a medium to coarse sand subsoil extending to a depth of 2 m. Coarse sand and gravel extend from 2 m to the water table at 6 m.

The lysimeters were filled first with subsoil, then with plow layer soil for the last 25 cm. As the soil was added, it was compacted by foot, and watered down. After filling, the soil settled less than 1 cm. At the time of this experiment the lysimeters had been filled for 1½ years.

Determination of Soil Water Conductivities and Diffusivities

As is shown later, the prediction of evaporation requires the soil water diffusivity characteristic for the surface layer, while drainage will be shown to be mainly a function of the soil water conductivity of the soil below the 25-cm depth. These parameters were determined in the laboratory in separate columns.

Soil water conductivities were determined by a method employed by Childs and Collis George (1950). If water is introduced at a constant rate into the top of a sufficiently long column, the moisture content and suction over a substantial length are uniform, so the potential gradient is only gravitational. The column was a 140-cm long lucite tube by 10-cm I.D., filled with the sand from the 25–60-cm depth packed to a bulk density 1.6 the value for the same layer in the field. Tensiometers made from 5-cm by 0.32-cm I.D. ceramic filter tubes were located at 30-, 50-, 70-, 90-, and 110-cm depths and connected to water manometers. Water was introduced at the top of the column at a constant flow rate using a small chromatography pump (Buchler Instruments, Inc., Fort Lee, N.J.). The pump was adjustable to provide flow rates from 20 to 1,000 ml/hour. For flow rates lower than 20 ml/hour, a cam timer was used to regulate the duty-cycle of the pump. The top of the column was covered with a polyethylene sheet which had a pin hole providing pressure equilibration with the atmosphere. Flow rate equilibrium through the column was obtained by observing when the tensiometers came to equilibrium and outflow at the bottom equalled inflow at the top. At this time, the matric suction was uniform throughout most of the column and the flow rate equalled the conductivity corresponding to the matric suction in the constant suction portion of the profile. This method was adequate for the range of drainage rates of interest in the sand.

Soil water diffusivities were determined in one of two ways depending on the moisture content. For volumetric water contents less than 0.12, the diffusivity was determined by measuring evaporation from lucite soil columns 10-cm long by 5-cm I.D. The diffusivity was calculated directly from

$$D(\theta) = 4L^2 \, (d\theta/dt)/\pi^2 \, (\theta - \theta_f) \qquad [1]$$

where L is the sample length, $d\theta/dt$ is the instantaneous rate of water loss, θ is the instantaneous volumetric water content, and θ_f is the final equilibrium volumetric water content of the sample (Gardner, 1956, 1962).

For volumetric water contents greater than 0.12, D was determined from the conductivity and specific water capacity using the definition, $D = -k(\theta) \, (d\phi_m/d\theta)$, where $k(\theta)$ is the conductivity (Fig. 2) and $(d\phi_m/d\theta)$ is the slope of the corresponding retention curve (Fig. 1).

RESULTS AND ANALYSIS

Soil Characteristics

Pertinent physical characteristics of Plainfield sand are shown in Fig. 1, 2, and 3. Matric suction, $-\phi_m$, as a function of volumetric water content, θ, for the 0–25, 25–60, and 60–100-cm depths is shown in Fig. 1. Capillary conductivity, k, as a function of matric suction for the 25–60-cm depth is shown in Fig. 2. In Fig. 3, soil water diffusivity, D, is shown as a function of volumetric water content for the 0–25-cm depth. The data are for the desorption part of the hysteresis cycle.

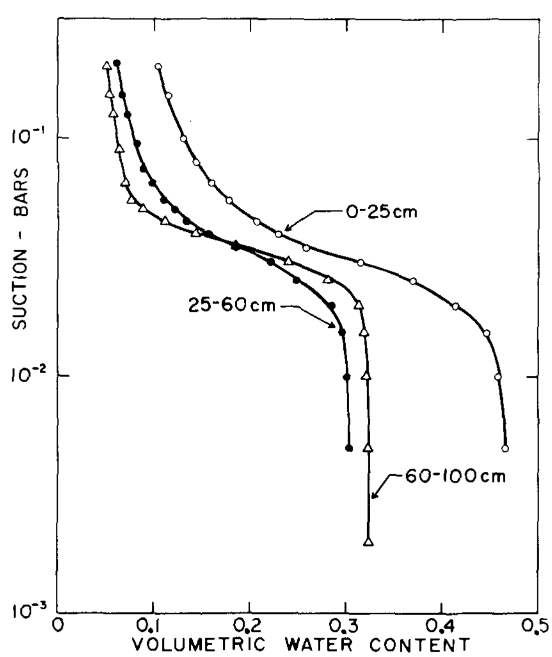

Fig. 1—Moisture retention characteristics of Plainfield sand for 0–25-cm, 25–60-cm, 60–100-cm depths.

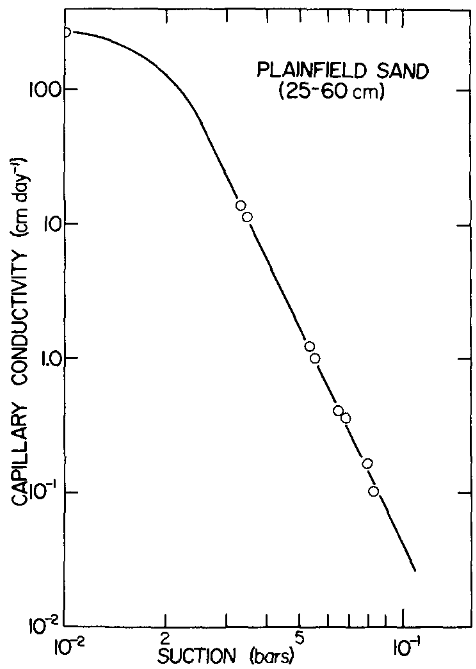

Fig. 2—Capillary conductivity as a function of soil-water suction for the 25–60-cm depth of Plainfield sand.

Evaporation

Evaporation was predicted using the solution of the unsaturated flow equation assuming one dimensional flow under isothermal conditions in a homogeneous soil profile

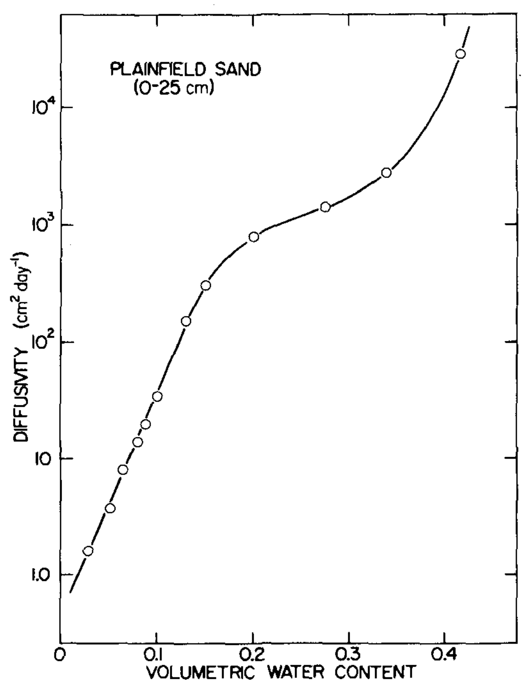

Fig. 3—Soil-water diffusivity as a function of volumetric water content for the 0–25-cm depth of Plainfield sand.

uniformly wet initially to infinite depth. A profile wet to a finite depth may be treated as semi-infinite in the initial stages of drying, or until about 50% of the water in the profile is evaporated (Gardner, 1959). The flow equation, initial and boundary conditons are:

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial x}\left[D(\theta)\,\frac{\partial \theta}{\partial x}\right] \qquad [2]$$

$$\theta = \theta_i,\ x \geq 0,\ t = 0$$

$$\theta = \theta_0,\ x = 0,\ t > 0$$

where θ is the volumetric water content, x is the distance, and $D(\theta)$ is the soil-water diffusivity. The validity of the assumption of isothermal conditions is subject to question, however, calculations by Philip (1957) show the temperature gradients to have only a modest influence. Hanks et al. (1967) showed in laboratory studies that the assumption of isothermal conditions lead to an error of no more than 10% for soils initially wet to near saturation.

Equation [2] can be solved analytically (Crank, 1956, p. 61) for a semi-infinite slab with constant diffusivity, D, to give the flux, q, at the boundary as,

$$q = (\theta_i - \theta_0)\ (D/\pi t)^{1/2} \qquad [3]$$

where θ_i is the initial water content, assumed constant for $t = 0$ and $x \geq 0$, θ_0 is the water content at the boundary ($x = 0$), assumed constant for $t > 0$. Integration of equation [3] with respect to time gives the cumulative flux, E.

$$E = 2(\theta_i - \theta_0)\ (Dt/\pi)^{1/2} \qquad [4]$$

If D depends on θ, equation [3] still holds where D is replaced by a weighted-mean diffusivity, $\bar{D}$.

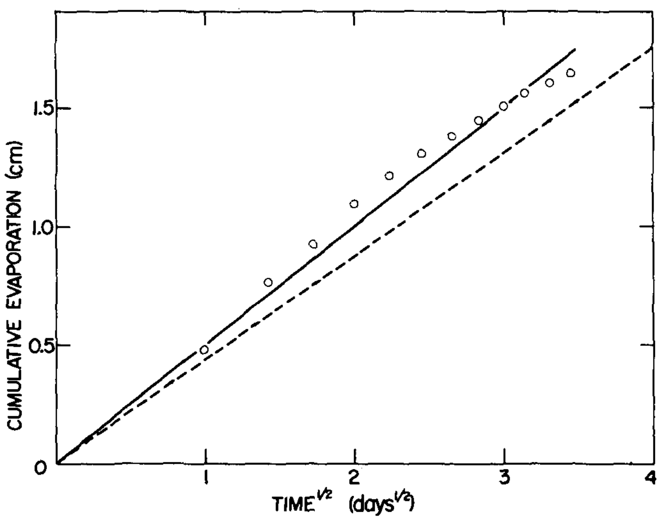

Fig. 4—Cumulative evaporation from bare Plainfield sand as a function of $t^{1/2}$. The circles are lysimeter data from Aug. 27 to Sept. 7, 1967. The solid line is the best fit by eye to the lysimeter data, while the dashed line is for $\bar{D} = 10$ cm^2/day obtained from laboratory data.

Crank (1956) also gives expressions relating the weighted-mean diffusivity to the true diffusivity. For desorption processes this relation is given by the integral:

$$\bar{D} = \frac{1.85}{(\theta_i - \theta_0)^{1.85}} \int_{\theta_0}^{\theta_i} D(\theta)\,(\theta_i - \theta)^{0.85}\,d\theta. \qquad [5]$$

For convenience a factor, C, is defined such that,

$$E = C\,t^{1/2} \qquad [6]$$

Thus

$$C = 2(\theta_i - \theta_0)\ (\bar{D}/\pi)^{1/2}. \qquad [7]$$

The constant, C, was determined from the data in Fig. 4 in which the cumulative evaporation for the drying period from Aug. 27 to Sept. 7 was plotted against $t^{1/2}$. This period was chosen because it was one of the longest without rain and because the lysimeter was attended each day to ensure a reliable estimate of the evaporation. The time, $t = 0$, was taken at the end of the preceding rainfall. A straight line was fitted to the points and the slope, C, was determined to be 0.496 cm/(day)$^{1/2}$. From equation [7], the weighted-mean diffusivity was 13 cm^2/day with $\theta_i - \theta_0$ equal to 0.12 cm^3/cm^3. The value of $\theta_i - \theta_0$ was obtained by assuming $\theta_0 \sim 0$ and taking θ_i to be the water content of the soil 2 days after a heavy rain. For 0–25-cm layer of Plainfield sand, θ_i was found experimentally to be 0.12. Departure from the $t^{1/2}$ relationship in the field data of Fig. 4 is apparent. In general, it is expected that evaporation after rainfall will depart from the $t^{1/2}$ relationship eventually because of the finite depth of wetting. This departure is the rule in the experiments of Gardner and Gardner (1969). However, the rate of evaporation from the Plainfield sand is so low that for the depths of wetting and the times involved during this experiment the soil profile behaved as though it were wet infinitely deep though departure from this behavior is evident after about 10 days.

Prediction of Cumulative Evaporation—Using the same value of C, equation [6] was applied at the beginning of each drying cycle. After each heavy rain, therefore, t was taken as zero. These theoretical cumulative evaporation values are the smooth curves in Fig. 5. The cumulative evaporation from the lysimeter is represented by the open circles. Dates and amount of irrigation or rainfall are indicated at the top of the figure.

On July 31, the lysimeter was covered for 8 days for calibration and the drainage system stopped. Thus for this period, daily evaporation, drainage and precipitation was zero. Moisture redistribution undoubtedly took place in the lysimeter during this period. This redistribution invalidates the assumptions made concerning the initial condition for the drying process. An exact solution of the equation for this case is difficult to obtain; however, it is possible to set limits on the evaporation. An upper limit of evaporation is represented by the upper curve in Fig. 5 which assumes that evaporation continued as $t^{1/2}$ as though there were no interruption. The lower curve represents the evaporation to be expected if no redistribution occurred

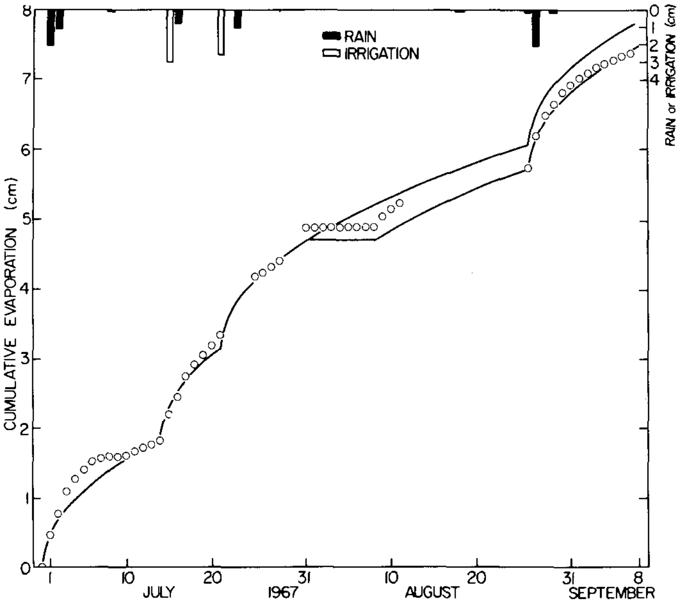

Fig. 5—Predicted cumulative evaporation from bare Plainfield sand compared with that measured by lysimeter. The circles are lysimeter data. The smooth curves were obtained using [6] with $C = 0.496$ cm day$^{-1/2}$ and setting $t = 0$ after each heavy rain.

at all and the net effect of the interruption was to translate the curve along the time axis a period of 8 days.

The total evaporation from June 30 to Sept. 7 measured by the lysimeter was 7.45 cm, while the predicted total was 7.50 cm, assuming no moisture redistribution during the covered period. Assuming possible moisture redistribution in the lysimeter, the predicted cumulative evaporation was 7.93 cm.

Calculation of C from Diffusivity Measurements—If the soil water diffusivity is known as a function of soil water content it is possible to evaluate the constant C in equation [6] independently.

The data plotted in Fig. 3 for $\theta < 0.15$, which is the usual case in the field, fall nearly enough on a straight line to assume an exponential relation between diffusivity and water content.

From Fig. 3 we find the diffusivity, D_0, at the lowest water content of the soil ($\theta_0 = 0$) to be about 0.5 cm^2/day. Experimentally θ drops to about 0.12 within 2 days. Using this value for θ_i gives a value for D_i of about 120 cm^2/day so that $\bar{D} = 10$ cm^2/day. Substituting the above values in [7] gives a value of C of 0.43 cm day$^{-1/2}$. This is about 13% lower than the value of 0.496 obtained from the lysimeter data. Equation [6] with $C = 0.43$ cm day$^{-1/2}$ is shown as the dashed line in Fig. 4. Considering the natural variability of soil and the difficulties of reproducing structure the agreement is reasonable.

Drainage in the Soil Profile

The precise prediction of drainage following irrigation of rainfall requires the solution of [2]. Wang and Lakshminarayana (1968) used a numerical technique to solve the problem of vertical drainage and infiltration. The results compared favorably with experimental data in Nielsen et al. (1964). Rubin (1968) solved numerically a two-dimensional problem of transient flow of water for unsaturated and partly saturated soils. Brutsaert et al. (1961) using an electrical analogue predicted water table drawdown from a series of steady state solutions. Young (1960) and Gardner (1962) have presented approximate solutions to the one-dimensional drainage problem. Gardner (1962) showed that solutions of the equation could be found which satisfied the boundary conditions and gave reasonable predictions of the drainage for times large enough so that initial conditions are not important.

The downward flux, $\partial F(z,t)/\partial t$, at time t and depth z may be expressed as,

$$\partial F\ (z, t)/\partial t = k(z, t)\ \partial \phi\ (z, t)/\partial z \qquad [8]$$

where $k(z, t)$ is the conductivity and ϕ is the total potential given by,

$$\phi = \phi_m + \phi_g \qquad [9]$$

where ϕ_m is the matric potential and ϕ_g is the gravitational potential. Expressing the potentials as heights of water in cm, and substituting [9] into [8] we have, in one dimension,

$$\partial F/\partial t = k(\partial \phi_m/\partial z + 1). \qquad [10]$$

In terms of the diffusivity, D, [10] can be written as,

$$\partial F/\partial t = D\ \partial \theta/\partial z + k. \qquad [11]$$

Data such as that of Prill et al. (1965) obtained from laboratory studies of drainage from various sands and Richards et al. (1956) for drainage in the field demonstrate that $D\ \partial\theta/\partial z$ is relatively small during a large fraction of the drainage process. Furthermore, it is observed that water drains almost equally from all layers above the initial wetting depth in a uniform profile. Because k increases rapidly with increasing θ, only an otherwise negligibly small increase in θ with depth accounts for the increase in flux with depth. Thus for drainage that is uniform with depth, k must increase linearly with depth. This may be expressed as,

$$k = \frac{1}{L}\ (\partial F/\partial t)\Big|_{z = L} \cdot z \qquad [12]$$

where L is the profile depth for which drainage is being calculated. While this relation holds over most of the profile, it does not apply in the vicinity of $z = 0$. The drainage rate at $z = L$ may be written:

$$\partial F/\partial t\Big|_{z = L} = f(S) \qquad [13]$$

where S is the profile water storage above the depth $z = L$.

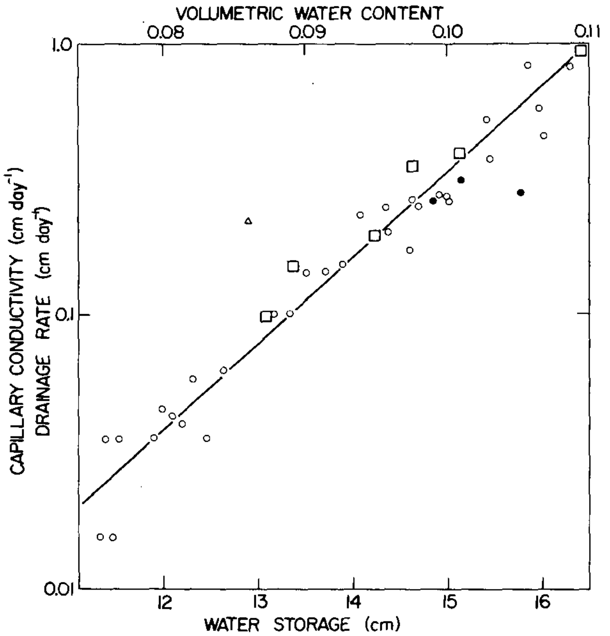

Fig. 6—Lysimeter drainage rate as a function of water storage (circles) and capillary conductivity of Plainfield sand as a function of soil water content (squares). The upper scale gives the soil water content corresponding to a given storage assuming a uniform water content distribution.

In Fig. 6, drainage rates measured during the year are plotted against corresponding lysimeter storages. The closed circles are data taken immediately following an application of 2.5 cm of irrigation water. They indicate that it takes about 2 days before increased water storage redistributes itself in the profile and beings to affect the drainage rate. The open-triangle data point was the first drainage value taken following an eight-day period during which the lysimeter was undergoing calibration tests with the drainage system off. The drainage rate was very nearly an exponential function of the storage. The equation of the line fitted to the data in Fig. 6 is,

$$dF/dt = 0.35 \exp\{0.70\ (S - 15.0)\} \quad [14]$$

where S is the storage in cm of an equivalent surface depth of water. Drainage is overestimated if the depth of wetting does not exceed the profile depth for which the drainage is being calculated, as is the case just after a small rainfall or shallow irrigation. In order to predict correctly the initial drainage a time lag can be introduced to allow time for redistribution of water so that the wetting front arrives at the depth $z = L$. In the present case, this is about 2 days. The error caused by neglecting this correction decreases with time and becomes negligible after 3 or 4 days.

Relationship of Drainage Rate to Capillary Conductivity—The values of capillary conductivity less than 1.0 cm day^{-1} in Fig. 2 are plotted against volumetric water content in Fig. 6 (open squares) by using the corresponding retention curve (25–60 cm) in Fig. 1. The volumetric water content scale at the top of Fig. 6 is also an average of the volumetric water content of the lysimeter obtained by dividing the water storage (cm) by the depth of the lysimeter (150 cm). The agreement between laboratory determined conductivities and field drainage rates at corresponding water contents verifies that in the normal range of drainage rates in the sand, dF/dt was determined primarily by the conductivity and the gravitational potential gradient.

At the lowest water contents observed during the experiment, the hydraulic gradient was at least ½ the gravitational gradient. Outside the lysimeter the gradient is more nearly 1 at all times in this soil in view of the depth of the water table. Thus, the normal profile may be expected to drain at only a slightly higher rate than the lysimeter.

Prediction of Water Storage and Drainage Rates

The storage, S_n, at the end of any day, n, was computed by adding to the storage, S_{n-1}, at the end of day (n-1), any recorded precipitation or irrigation, ΔP_n, and subtracting predicted drainage, ΔF_n, and predicted evaporation, ΔE_n, for day n. This is expressed by the hydrologic equation written as,

$$S_n = S_{n-1} + \Delta P_n - \Delta E_n - \Delta F_n \quad [15]$$

where $n = 1, 2, 3, \ldots$, the number of days from starting the prediction. An initial storage, S_0, ($n = 1$) is required to begin the procedure. This is the estimated or measured amount of water in the profile at the beginning of the period of interest. The daily evaporation, ΔE_n, is the difference between two successive values of cumulative evaporation, E_n, computed from [6] with $C = 0.496$ cm day$^{-1/2}$. The daily drainage, ΔF_n, is obtained from,

$$\Delta F_n = (\Delta F/\Delta t)_n \Delta t \quad [16]$$

where $\Delta t = 1$ day, and $(\Delta F/\Delta t)_n$ is computed from [14] written in the form,

$$(\Delta F/\Delta t)_n = 0.35 \exp\{0.70\ (S_{n-1} - 15.0)\}. \quad [17]$$

The drainage on day n was computed on the basis of the storage at the end of day ($n - 1$). Predicted drainage rates

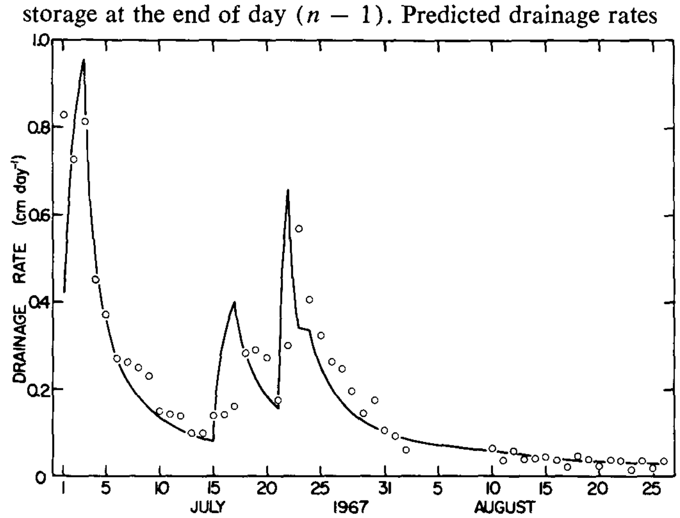

Fig. 7—Predicted drainage (solid line) from bare Plainfield sand compared with that measured by lysimeter (circles).

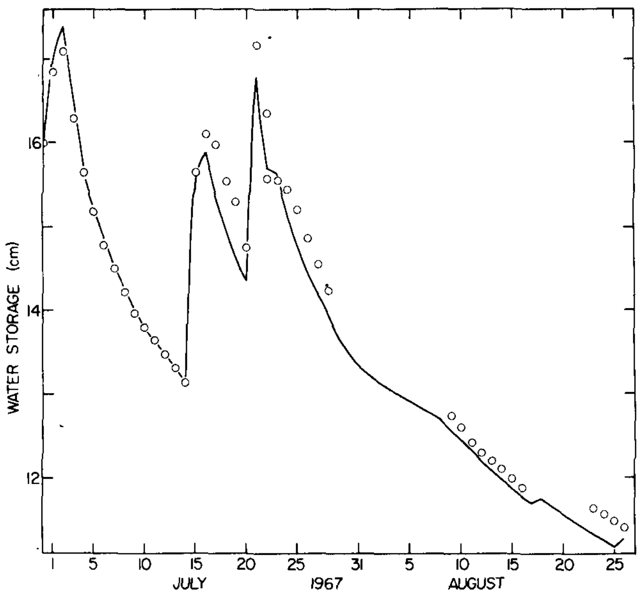

Fig. 8—Predicted soil water storage (solid line) in bare Plainfield sand compared with that measured by lysimeter (circles).

and those measured by the lysimeter are compared for the period June 30 to Aug. 26 in Fig. 7.

The storage predicted using [15] is compared with lysimeter storage values in Fig. 8 for the period June 30 to Aug. 26. The measured storage value at the end of June 30 (16.0 cm) was used as the initial storage value (S_0) of the predicted curve. At the end of the test period, predicted and measured storage differed by about 0.3 cm.

For the period July 1 to Sept. 12, there were 13.2 cm of precipitation and irrigation, while drainage was 10.6 cm and evaporation was 7.5 cm. Water storage decreased by 4.9 cm, from 16.0 at the end of June 30 to 11.1 cm on Sept. 12. These high drainage and low evaporation values are consequences of the small diffusivities at low moisture contents and large conductivities at high moisture contents of Plainfield sand.

DISCUSSION AND CONCLUSION

In this experiment on a bare soil, the application of simplified flow theory to both the evaporation and drainage processes provided a method of predicting water storage in the soil profile. By making a simple square root of time approximation, cumulative evaporation was estimated to within 5%.

The cumulative evaporation constant, C, used in equation [6] can be obtained directly from one or two drying periods using a lysimeter. More conveniently, C can be calculated from equation [7] using estimates of θ_i and θ_0 if soil-water diffusivities have been determined.

It should be emphasized that the analysis of the data for this particular system represents what is probably a very unusual system. Evaporation from finer textured soils cannot in general be described by such a simple expression, as is obvious from the data by Gardner and Gardner (1969). The drainage situation will be quite different in layered soils and in soils in which the water table is near enough to the surface to be a factor. However, it is believed the general approach may apply to a broader range of soil types. That is, it may be possible to describe evaporation as a function of time or water content and drainage as a simple function of profile storage for purposes of calculating a water budget. A more complete analysis will be required in order to specify the actual water content distribution but it is a characteristic of the unsaturated flow equation that fluxes into and out of a system may be estimated with surprising precision using very gross approximations such as substituting a weighted-mean diffusivity for the actual diffusivity. Experiments on a wide range of soil textures will be required to determine the general applicability of this approach.

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