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[4]

CONSTANT-RAINFALL INFILTRATION

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ABSTRACT

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An approximate solution of the infiltration equation for rain of constant intensity is developed. From the theory, the ponding time and the development of the moisture profile at time shorter than ponding time are determined. Using the theoretical background, simple empirical and algebraic equations are derived for the calculation of the ponding time. The solutions are compared with the results of the numerical analysis; good agreement has been found. The solution of the infiltration for time greater than ponding time is obtained either for "delta function" soil or for an empirical infiltration equation.

INTRODUCTION

The practical importance of studies on rain infiltration has been recognized for a long time and numerical solutions have been developed to describe the phenomena (Rubin and Steinhardt, 1963, 1964; Rubin, 1966). In addition, Parlange (1972) published an approximate solution of the infiltration equation with a constant flux at the surface, while Mein and Larson (1973), and Swartzendruber (1974) discussed important features of the problem for "delta function" soil. However, a detailed discussion comparing an approximate analytical solution with the results of numerical analysis and with some intuitive or empirical approaches is still lacking. Furthermore, some hydrologically important terms such as "ponding time" are often not exactly interpreted.

The present paper attempts to identify explicitly the important aspects of the process, and develops an approximate analysis of constant-rate rain infiltration in a homogeneous soil.

APPROXIMATE ANALYTICAL SOLUTION

The basic equation describing one-dimensional infiltration can be written:

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$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z}\left[D\frac{\partial \theta}{\partial z}\right] - \frac{dk}{d\theta}\frac{\partial \theta}{\partial z} \tag{1}$$

and can be solved for the initial condition:

$$t = 0; \quad \theta = \theta_i; \quad z > 0 \tag{2}$$

where θ is the volumetric moisture content; t is time; z is the vertical co-ordinate positive downward; D is the moisture diffusivity, $D(\theta)$; k is the unsaturated hydraulic conductivity, $k(\theta)$, reaching k_s at the saturation.

However, the boundary condition depends on the nature and duration of the rainfall intensity v_r, relative to v_c, the steady long-time infiltration rate in a ponded soil with hydraulic head $\psi_0 \to 0$. Two conditions are recognised:

(1) $0 < v_r/v_c < 1$. In this case the flow is described by a constant-flux boundary condition, viz.:

$$t \geqslant 0, \quad D(\partial\theta/\partial z) - k = -v_r, \quad z = 0 \tag{3}$$

and the surface water content θ_0 is time-dependent, approaching after great period θ_{0c}, depending on $v_r = k(\theta_{0c}) - k(\theta_{0i})$. The soil surface never ponds.

(2) $v_r/v_c > 1$. Two time intervals now become important:

(a) $0 < t < t_p$. During this interval, the rate at which the soil can accept water exceeds v_r and the soil surface remains unsaturated. The flow process is described by condition (3).

(b) $t \geqslant t_p$. At the ponding time t_p the soil surface is effectively saturated the rainfall rate exceeds then the rate at which the soil will accept water, the excess runs off and infiltration can be described in terms of the constant concentration boundary condition:

$$t \geqslant t_p, \quad \theta = \theta_s, \quad z = 0 \tag{4}$$

If $v_r/v_c \to \infty$, the flow is characterized by the constant-concentration boundary condition, viz.:

$$t > 0, \quad \theta = \theta_s, \quad z = 0 \tag{4a}$$

where θ_s is the moisture content at saturation.

For the solution of the cases (1) and (2a), i.e. for the constant-rate infiltration, the concept of flux concentration relation (Philip, 1973) will be applied. This approach improves substantially the Parlange's (1971) original perturbation method. The flux concentration relation, $F(\Theta)$, is defined by:

$$F(\Theta) = (v - k_i)/(v_r - k_i) \tag{5}$$

where v is the flow rate, k_i is the hydraulic conductivity at $\theta = \theta_i$, and the relative moisture content:

$$\Theta = (\theta - \theta_i)/(\theta_0 - \theta_i)$$

Because θ_0 is time-dependent within the time interval $0 < t < t_p$, F will also be time-dependent, i.e. $F(\Theta,t)$. The further procedure follows the derivation of the solution of eq. 1 by Smiles (1978) for a similar problem of the constant-flux filtration in a two-phase system of slurry.

Eq. 1 is the result of the combination of the flow equation:

$$v = -D(\partial\theta/\partial z) + k \tag{6}$$

and of the continuity equation:

$$\partial\theta/\partial t = -\partial v/\partial z \tag{7}$$

These equations are now to be treated separately. Introducing eq. 5 into eq. 6, we obtain:

$$F(v_r - k_i) - (k - k_i) = -D(\partial\theta/\partial z) \tag{8}$$

and the integration from $z=0$ yields:

$$z = \int_{\theta}^{\theta_0(t)} \frac{D}{F(v_r - k_i) - (k - k_i)} \, d\theta \tag{9}$$

see also equation (26) of Philip and Knight (1974). Then, integration of eq. 7 gives for $v_i = k_i$:

$$(v_r - k_i)t = \int_0^z \theta \, dz - \int_0^z \theta_i \, dz \tag{10}$$

Substitution of eq. 9 into eq. 10 and integration leads to:

$$(v_r - k_i)t = \int_{\theta_i}^{\theta_0(t)} \frac{(\theta - \theta_i)D}{F(v_r - k_i) - (k - k_i)} \, d\theta \tag{11}$$

see also equation (14a) of Smiles (1978).

Note that eq. 9 corresponds with Parlange's (1972) equation (8) with $F = \Theta$. This $F(\Theta)$ relationship restricts principally the validity of the Parlange's solution either to the "delta function" soil for all t, or to all soils for $t \to \infty$ (Philip, 1973). However, in our problem with $v_r > k_s$, the condition of $t \to \infty$ is not applicable. On the other hand, Parlange's (1972) equation (6) is the analogue of our eq. 11, when we set $F=1$, a condition not applicable to infiltration. The problems of Parlange's method of solution of constant-concentration infiltration are discussed in detail in the paper of Knight and Philip (1974).

Therefore, we consider the derived solutions as more general ones, fitting all soil models. If $F(\Theta)$ is chosen properly, the iteration procedure, originally proposed for this type of solution by Philip and Knight (1974), appears to be unnecessary at least for the majority of practical tasks where slight errors can be neglected.

Comparison with the numerical solutions

In order to check up the analytical solution, the results of Rubin's (1966) numerical procedure for infiltration from constant rain intensity in Rehovot sand were compared with the results obtained from our solution. The gradual increase of moisture content on the soil surface $\theta_0(t)$ was computed using eq. 11, and the moisture profiles $\theta(z)$ at $t = 20$ s and $t = 110$ s, respectively, were computed from eq. 9. In both eqs. 9 and 11, the functional relationship $F(\Theta)$ was simplified by the approximate expression $F = \Theta$ which is valid exactly for "delta function" soil and to which the characteristics of the Rehovot sand are assumed to be relatively close. For better understanding of the importance of $F(\Theta)$, the relationship $F = 1$ (Parlange's solution) was also included. In the computation, Rubin's (1966) analytical expressions of moisture potential, $\psi(\theta)$; his equation (41), $k(\theta)$; and his equation (42) were used. Then the $D(\theta)$ relationship was obtained from $k(\theta)$ and from the derivative of $\psi(\theta)$ with respect to θ. The values of ψ, k and D corresponding to the series of moisture contents θ are arranged for the sake of convenience in Table I. The constant intensity rain was taken at $v_r = 1.5k_s = 0.01995$ cm s^{-1}.

The increase of the moisture content on the surface with time t is plotted in Fig. 1. It can be seen that there is a rapid change of moisture content close to $t = 0$, while near to $t = t_p$, the change of θ_0 is very small. From this it follows that the experimental determination of t_p at $\theta_0 = \theta_s$ will be very difficult. Parlange's procedure with $F = 1$ leads to higher values of θ_0 and consequently to the underestimation of the ponding time t_p.

In Fig. 2, the comparison between Rubin's (1966) data and the computations of the moisture profile $\theta(z)$ according to eq. 9 is plotted for times $t = 20$ s and $t = 110$ s. For shorter periods, the choice of F does not play an important role and there is a very good agreement between the results of Rubin's numerical

TABLE I

Values of ψ — moisture potential per unit weight; k — unsaturated conductivity; and D — diffusivity, depending upon water content θ for Rehovot sand of Rubin (1966)

θ (cm^3 cm^{-3})	ψ (cm)	k (cm s^{-1})	D (cm^2 s^{-1})
0.01	$-2.82 \cdot 10^5$	$2.95 \cdot 10^{-13}$	$3.12 \cdot 10^{-5}$
0.05	-75	$6.12 \cdot 10^{-6}$	$7.61 \cdot 10^{-3}$
0.10	-43	$6.39 \cdot 10^{-5}$	$2.06 \cdot 10^{-2}$
0.15	-32	$1.65 \cdot 10^{-4}$	$2.45 \cdot 10^{-2}$
0.20	-26	$4.18 \cdot 10^{-4}$	$3.83 \cdot 10^{-2}$
0.25	-22	$1.06 \cdot 10^{-3}$	$7.49 \cdot 10^{-2}$
0.30	-19	$2.69 \cdot 10^{-3}$	$1.82 \cdot 10^{-1}$
0.35	-15	$6.81 \cdot 10^{-3}$	$5.40 \cdot 10^{-1}$
0.38	-13	$1.19 \cdot 10^{-2}$	1.12
0.387	-12	$1.33 \cdot 10^{-2}$	1.33

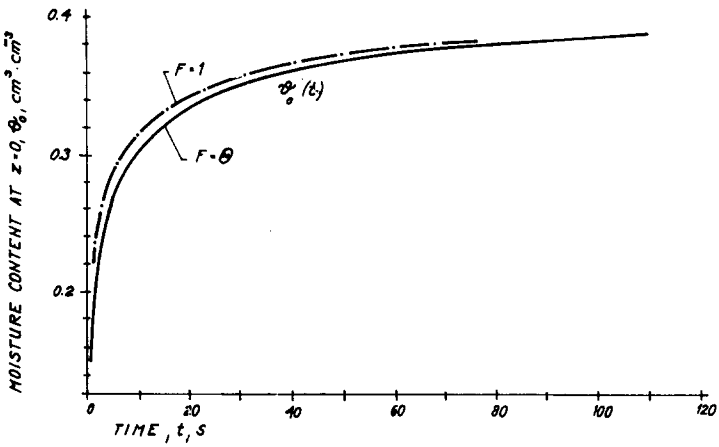

Fig. 1. The increase of the moisture content on the surface with time $\theta_0(t)$ during the rain infiltration according to eq. 11 for Rubin's (1966) Rehovot sand with constant rain intensity $v_r = 1.5k_s = 0.01995$ cm s^{-1}.

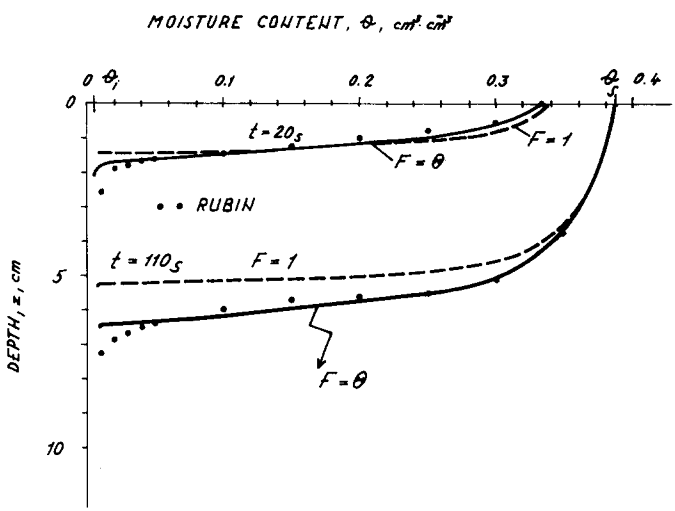

Fig. 2. The moisture content profiles $\theta(z)$ during rain infiltration at time t = 20 s and t = 110 s according to eq. 9 for Rubin's (1966) Rehovot sand and for rain intensity v_r = 0.01995 cm s^{-1}. *Full points* correspond to Rubin's data of numerical analysis.

procedure and the computed profile according to our solution, eq. 9 for $F = \Theta$, while for Parlange's F=1, a difference occurs. For longer periods, practically close to the ponding time, the results are sensitive against the approximation of F and it can be seen that a good agreement is obtained for $F = \Theta$, which corresponds to the "delta function" soil, while Parlange's procedure for F=1 leads to a different moisture profile.

In a similar way, the results of the numerical analysis of Haverkamp et al. (1977) were used. In their model, saturated hydraulic conductivity of sand was $k_s = 0.0944$ cm s^{-1} and the constant flux on the boundary was $v_r = 0.0038$ cm s^{-1} $= 0.403k_s$. Moisture profiles computed according to eq. 9 are compared with Haverkamp's data in Fig. 3. A good agreement was found between both. Let us remind here that a relatively poorer agreement between Parlange's procedure and the results of the numerical analysis were demonstrated in the paper of Haverkamp et al. The sensitivity of our solution upon the approximate estimation of $F(\Theta)$ can be seen, but the differences between the profiles for $F = \Theta$ and $F = \Theta^{0.8}$ are of low practical importance. Generally, it can be concluded that the proposed analytical solution leads to a good agreement with the results of numerical analysis, and that the approximate estimation of $F \approx \Theta^m$ is suitable for practical tasks. However, a more detailed study on the approximate expression for $F(\Theta)$ would be advantageous.

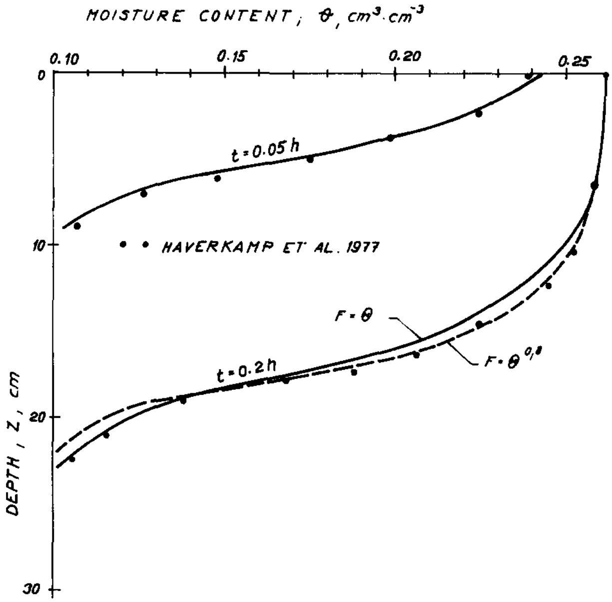

Fig. 3. The moisture content profiles $\theta(z)$ during rain infiltration at time $t = 0.05$ hr. and $t = 0.2$ hr. according to eq. 9 for Haverkamp et al.'s (1977) sand and for $v_r = 0.0038$ cm s^{-1}. *Full points* are Haverkamp et al.'s results of the numerical analysis.

PONDING TIME

Theory

The term ponding time t_p denotes the time at which the rainfall rate exceeds the rate at which the soil surface can accept water so that water commences to pond on the surface. Since ponding time cannot occur before θ_0 reaches saturation, the condition for $t = t_p$ is $\theta_0 = \theta_s$. Applying this to eq. 11, neglecting gravity and if $v_r > k_s$, we obtain:

$$t_p = \frac{1}{v_r^2} \int_{\theta_i}^{\theta_s} (\theta - \theta_i) \frac{D}{F} \, d\theta \tag{12}$$

The nth iterative estimate of sorptivity S_n is [Philip and Knight, 1974, equation (14)]:

$$S_n = \left[2 \int_{\theta_i}^{\theta_s} (\theta - \theta_i) \frac{D}{F_n} \, d\theta \right]^{1/2} \tag{13}$$

where F_n is the nth iterative estimate of F. If F is carefully chosen, the iteration procedure can be neglected, i.e. $F_n \approx F(\Theta)$, and then:

$$t_p = S^2/2v_r^2 \tag{14}$$

It is useful to compare eq. 14 with the results of two intuitive procedures. For this comparison, the equation of infiltration rate for the constant-concentration boundary condition (4a) will be applied and index p can be used. If gravitation is neglected for the sake of simplicity, then:

$$v_p = \tfrac{1}{2} S t^{-1/2} \tag{15}$$

In the first alternative, the ponding time is identified with the time when $v_p = v_r$. It follows from the eq. 15 that $t_{pv} = S^2/4v_r^2$.

In the second alternative, the ponding time is taken as equal to the time when $i_p = i_r$, where i designates the cumulative value either of the constant-concentration infiltration (index p), or of the rain (index r):

$$t_{pi} v_r = \int_0^{t_{pi}} v_p \, dt \tag{16}$$

as demonstrated in Fig. 4. Using eq. 15 we obtain $t_{pi} = S^2/v_r^2$. The intuitively derived values of ponding time t_{pv} and t_{pi} are in the following relation to ponding time t_p obtained by quasi-analytical solution, when gravitation is neglected: $t_p = 2t_{pv} = t_{pi}/2$.

It is obvious that the same result as in eq. 14 will be obtained if eq. 15 is substituted in:

$$v_r t_p = \int_0^{t_{pv}} v_p \, dt \tag{17}$$

where $t_{pv} = S^2/4v_r^2$, as derived earlier. The relation (17) will be utilized later on. Therefore, the value of t_p can also be obtained graphically, using eq. 17, as demonstrated in Fig. 4. This procedure is applicable more generally to the determination of the ponding time even from a rain of non-constant intensity.

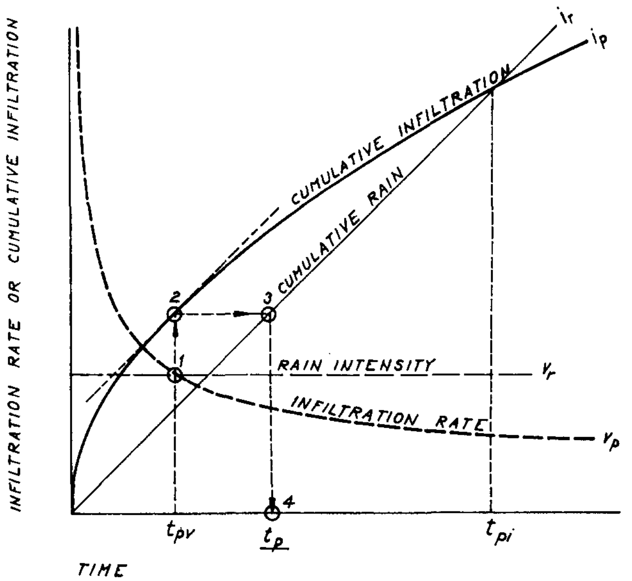

Fig. 4. Determination of ponding time t_p from the graphs of infiltration with constant-concentration boundary condition and from the graphs of rainfall. Graphs of both determinations are plotted as rate vs. time and cumulative values vs. time.

The relation (17) has been extended to a general formulation of the dependence of the rate of rain infiltration upon the cumulative infiltration by Mls (1980).

When gravitation is considered, eq. 11 will be used for $t = t_p$ and $\theta_0 = \theta_s$ again, giving:

$$t_p = \frac{1}{v_r - k_i} \int_{\theta_i}^{\theta_s} \frac{(\theta - \theta_i)D}{F(v_r - k_i) - (k - k_i)} \, d\theta \qquad (18)$$

and the t_p value will be shifted slightly nearer to the t_{pv} value, as can be seen in Fig. 4.

Let us remind here that Parlange's (1972) approximate equation leads to:

$$t_p = \int_{\theta_i}^{\theta_s} \frac{(\theta - \theta_i)D}{v_r} \, \frac{1}{v_r - k} \, d\theta \qquad (19)$$

If k_i in eq. 18 is negligibly small, we find the identity of eqs. 18 and 19 for F=1. Since $F < 1$ with the exception of $\Theta = 1$, the solution according to eq. 19 would lead to a systematically lower value of t_p, when compared with eq. 18.

However, it is a common situation that $D(\theta)$ and $k(\theta)$ are not known and the only information at hand is of infiltration with the constant-concentration boundary condition, i.e. the result of the double ring infiltrometer test. The test is then evaluated usually according to one of the following equations:

$$v_p = St^{-1/2}/2 + A \qquad \text{(Philip, 1957)} \qquad (20)$$

$$v_p = C_1 t^{-\alpha} \qquad \text{(Kostiakov, 1932)} \qquad (21)$$

$$v_p = C_2 t^{-\beta} + v_c \qquad \text{(Mezencev, 1948)} \qquad (22)$$

$$v_p = (v_1^* - v_c^*)t^{-\beta} + v_c \qquad \text{(Dvořák and Holý, 1960)} \qquad (23)$$

$$v_p = k_s(\psi_0 - \psi_f + L_f)/L_f \qquad \text{(Green and Ampt, 1911)} \qquad (24)$$

where A is a constant close to hydraulic conductivity k_s; C_1, C_2, α, β are empirical constants with appropriate dimensions $\alpha < 1$, $\beta < 1$; v_1 is the infiltration rate at $t=1$, the most appropriate unit is the minute; the asterisk denotes the changed dimensions with regard to the empirical coefficient β; ψ_0 is the moisture potential at $z = 0$, expressed per unit weight, i.e. [L], here $\psi_0 \geqslant 0$; ψ_f is the moisture potential on the wetting front, numerically negative; and L_f is the depth of the wetting front.

Combining eq. 20 with eq. 17, we obtain for $v_r = bA$:

$$t_p = \left(\frac{S}{A}\right)^2 \frac{2b-1}{4b(b-1)^2} \qquad (25)$$

The graphical interpretation of eq. 25 is shown in Fig. 5. It can be seen that the decrease of the sorptivity causes a reduction of the ponding time of more than the same order, e.g., if the change of the initial moisture content from wilting point to field capacity causes the decrease of the sorptivity by roughly half an order of magnitude, the ponding time will be reduced by more than

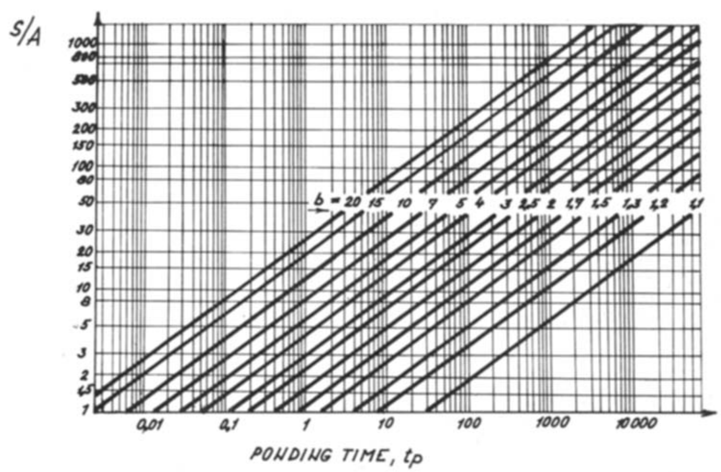

Fig. 5. The dependence of the ponding time t_p upon S/A ratio, where S is the sorptivity, A the second term in Philip's (1957) algebraic equation of infiltration for rain intensities $v_r = bA$, see eq. 25.

half an order. The relationship between the ponding time and the rain intensity is strongly non-linear, a great decrease of the rain intensity within the range of high intensities causes a slight increase of the ponding time while in the range of low intensities a small decrease of the intensity results in great increase of the ponding time, just as follows from eq. 25.

Mezencev's equation, in the form of eq. 23, gives with eq. 17 for $v_1 = av_c$ and $v_r = bv_c$ the expression:

$$t_p = \left[\frac{a-1}{b-1}\right]^{1/\beta} \frac{b-\beta}{b(1-\beta)} \tag{26}$$

which is an equivalent of eq. 25 for $\beta=1/2$, $S=2(v_1 - v_c)$, $A=v_c$. An equation similar to eq. 26 was intuitively proposed by Benetin (1970). Eq. 26 is also the solution of Kostiakov's equation (21) for $v_c=0$.

For the sake of completeness, the solution of t_p for eq. 24 (see Mein and Larson, 1973) will also be included, giving for $v_r=bk_s$:

$$t_p = -\frac{\psi_f(\theta_s - \theta_i)}{k_s} \frac{1}{b(b-1)} \tag{27}$$

The influence of the initial moisture content θ_i upon the value of t_p for the variation of v_r can be read from the graph in Fig. 6. It is demonstrated here that the influence of the initial moisture content θ_i, or, generally of $(\theta_s - \theta_i)$ decreases with the decrease of the rain intensity and that the influence of θ_i will be reduced in sandy soils with high k_s and low ψ_f values.

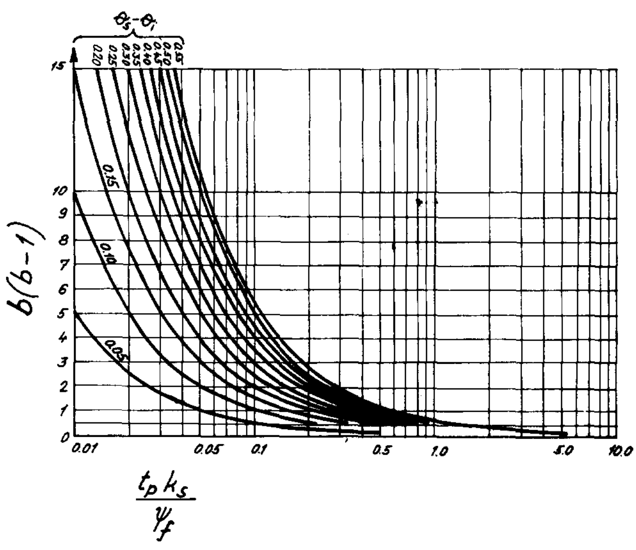

Fig. 6. The ponding time t_p as influenced by the initial moisture θ_i or by the moisture complement $(\theta_s - \theta_i)$ and by the rain intensity $v_r = bk_s$ according to eq. 27 derived for "delta function" soil.

Comparison with the results of the numerical analysis

The solution of the ponding time t_p according to eq. 26 is compared with the results obtained by the numerical analysis of Smith (1972) and with the corrected results obtained subsequently by Parlange and Smith (1976) for the

TABLE II

Comparison of ponding time t_p determined numerically (Smith, 1972; Parlange and Smith, 1975) with the analytical solution according to eq. 26 and with the values of t_{pv} (eq. 28) and t_{pi} (eq. 29), see also Fig. 4

Smith's data for infiltration if $v_r \to \infty$	v_r (cm/min.)	Numerically		Analytically		
		1972 t_p (min.)	1975 t_p (min.)	eq. 28 t_{pv} (min.)	eq. 26 t_p (min.)	eq. 29 t_{pi} (min.)
Poudre sand	0.212	35.0		26.62	39.41	119.69
v_c = 0.1397 cm/min.	0.339	9.05	8.74	4.70	8.60	21.15
$v_1 - v_c$ = 0.493 cm/min.	0.423	5.2	5.09	2.58	5.01	11.59
β = 0.585	0.508	3.52	3.36	1.65	3.33	7.40
	0.635	2.13	2.07	0.99	2.08	4.46
	0.762	1.45	1.39	0.67	1.44	3.02
	0.931	0.93	0.88	0.44	0.98	2.00
Nickel gravelly	0.0847	13.07	17.13	8.78	17.13	39.26
sandy loam	0.127	5.73	7.17	3.42	7.17	15.30
v_c = 0.0267 cm/min.	0.1481	4.16	5.18	2.46	5.26	11.01
$v_1 - v_c$ = 0.205 cm/min.	0.1693	3.14	3.87	1.87	4.05	8.35
β = 0.581	0.1905	2.44	2.97	1.47	3.23	6.58
	0.2117	1.94	2.32	1.19	2.64	5.33
Nibley silty clay	0.0635	33.41	30.12	16.53	31.72	71.09
loam	0.0868	17.42	16.48	7.98	16.02	34.33
v_c = 0.0167 cm/min.	0.127	7.83	6.92	3.53	7.35	15.17
$v_1 - v_c$ = 0.222 cm/min.	0.148	5.73	4.94	2.58	5.43	11.08
β = 0.555	0.169	4.33	3.80	1.97	4.19	8.48
	0.191	3.39	3.02	1.55	3.31	6.65
	0.212	2.78		1.26	2.71	5.42
Colby silt loam	0.0635	13.42	11.8	6.40	12.82	26.84
v_c = 0.0085 cm/min.	0.0847	7.58	6.85	3.49	7.12	14.62
$v_1 - v_c$ = 0.149 cm/min.	0.1058	4.83	4.33	2.21	4.57	9.28
β = 0.537	0.1270	3.32	2.92	1.53	3.19	6.43
	0.1693	1.79	1.57	0.87	1.82	3.64
	0.3175	0.59	0.483	0.26	0.55	1.08
Muren clay	0.0817	15.64	15.71	8.09	16.62	34.22
v_c = 0.0095 cm/min.	0.1270	7.04	7.12	3.56	7.46	15.04
$v_1 - v_c$ = 0.234 cm/min.	0.1481	5.21	5.25	2.62	5.54	11.10
β = 0.543	0.1693	4.02	4.06	2.02	4.28	8.54
	0.1905	3.19	3.22	1.61	3.42	6.79
	0.2138	2.61	2.54	1.28	2.74	5.43

same entry values of soils and rain intensities. The data are presented in Table II. In addition, this table contains the values of t_{pv} and t_{pi} so that the intuitive approaches can be compared with the relation (17) which was the basis for obtaining eq. 26. Here again, t_{pv} is the time of intersection of the infiltration rate for boundary condition (4a) with the rain intensity, i.e. for $v_p = v_r$ by using eq. 23:

$$t_{pv} = [(a-1)/(b-1)]^{1/\beta} \tag{28}$$

and t_{pi} is the time of intersection of the cumulative infiltration for boundary condition (4a) with the cumulative rain, i.e. for $i_p = i_r$ from eq. 23:

$$t_{pi} = [(a-1)/\{(b-1)(1-\beta)\}]^{1/\beta} \tag{29}$$

where $a = v_1/v_c$, $b = v_r/v_c$. It can be seen from Table II, that the t_p values calculated according to eq. 26 were very close to the numerically obtained t_p values of Smith (1972) and even closer to the ones of Parlange and Smith (1975). As it follows from comparison, t_p value is in between those of t_{pv} and t_{pi}. However, the information on mutual relationships derived for negligible gravity is not very exact.

INFILTRATION IN THE TIME INTERVAL $t > t_p$

For $t > t_p$ we suppose that there is no water accumulation on the soil surface, the runoff conducts all the excess water away and at $z = 0$ the condition $\psi_0 = 0$ is maintained. The flow is therefore characterized by the constant-concentration condition on the boundary with the appropriate shift of the t-axis.

For "delta function" soil, it follows from eq. 24 that:

$$k_s \int_{t_p}^{t>t_p} dt = \int_{i_p}^{i>i_p} \frac{i}{i - \psi_f(\theta_s - \theta_i)} di \tag{30}$$

Solving and re-arranging eq. 30, we obtain:

$$t^* = i_I^* - \ln(1 + i_I^* - i_{II}^*) \tag{31}$$

where the asterisk denotes the dimensionless terms:

$$t^* = -k_s t/[(\theta_s - \theta_i)\psi_f]$$

$$i_I^* = i^* - k_s/v_r, \qquad i_{II}^* = i^* k_s/v_r$$

and

$$i^* = -i/[(\theta_s - \theta_i)\psi_f]$$

Eq. 31 will be compared with the solution of eq. 24 for ponded infiltration (i.e. for constant-concentration boundary condition) when $\psi_0 = 0$ for $t \geqslant 0$:

$$t^* = i^* - \ln(1 + i^*) \tag{32}$$

The condition for which eq. 32 was derived can also be formulated as infiltration from the rain of intensity $v_r \to \infty$, when the non-infiltrated excess water is removed by the instantaneous runoff. If in eq. 31 $v_r = \infty$, then eq. 32 is obtained.

The use of eq. 31 is difficult due to the implicit expression of the cumulative infiltration. The eqs. 20 and 23 will be thereafter further applied. If the infiltration rate from the rain at $t = t_p$ equals the rate of infiltration with constant-concentration boundary condition at t_{pv}, we are allowed to assume that eqs. 20 and 23 will remain valid provided that the time scale is shifted by $(t_p - t_{pv})$. The eq. 23 can be transformed to:

$$v = v_c + (v_1^* - v_c^*)\left[t - \left(\frac{a-1}{b-1}\right)^{1/\beta}\left(\frac{b-\beta}{b(1-\beta)} - 1\right)\right]^{-\beta} \tag{33}$$

and eq. 20 to:

$$v = \tfrac{1}{2}S\left[t - \frac{S^2}{4A^2 b(b-1)}\right]^{-1/2} + A \tag{34}$$

Eq. 33 was used for Smith's (1972) data and some of the evaluated curves of the infiltration rate vs. time are plotted in Fig. 7 as an example of a very close agreement between the numerical analysis and the application of eq. 33. The rainfall infiltration can be estimated when the results of the simple field test with double ring infiltrometer are known. These conclusions are valid for

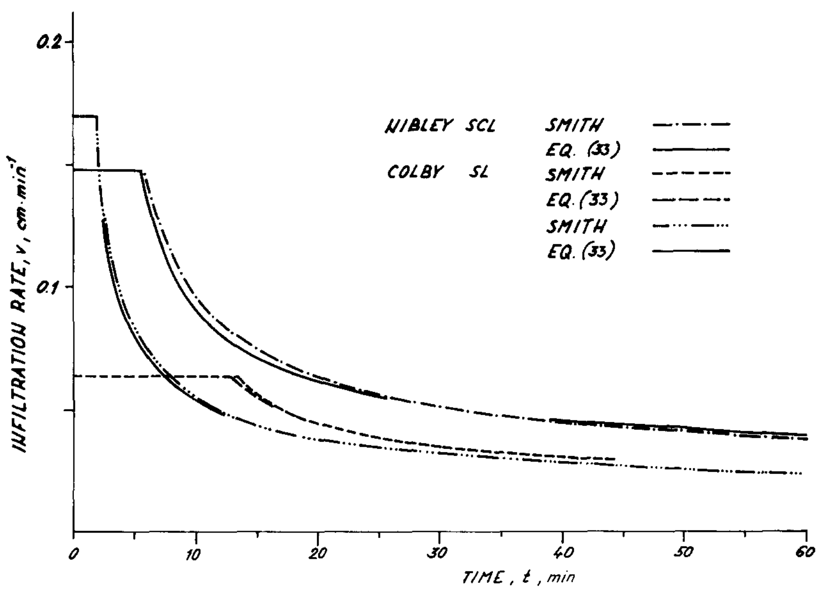

Fig. 7. Comparison of infiltration rate from rainfall according to Smith's (1972) numerical procedure and according to eq. 33 when information on the ponded infiltration is given.

homogeneous soils without existence of preferential ways, cracks and other types of heterogeneity and irregularity in the porous system, see Kutílek and Novák (1976), and Peschke and Kutílek (1976).

CONCLUSIONS

The quasi-analytical technique of solution of infiltration by Philip and Knight (1974) was applied to infiltration with constant flux through the boundary. This type of flow is the simplest one for rainfall infiltration and for hydrology. The results of the analytical procedure were compared with empirical approaches and with the numerical solutions of Rubin (1966), Smith (1972), and Haverkamp et al. (1977).

Ponding time t_p can be computed when the basic transport coefficients $D(\theta)$ and $k(\theta)$ of the soil are known, see eq. 18. Or, if the infiltration rate—time relationship for infiltration with constant-concentration boundary condition is measured and the rain intensity v_r is given, the t_p value can simply be determined according to the general eq. 17 which is further elaborated in eq. 25, eq. 26, or eq. 27.

For the computation of the moisture profile when $t < t_p$, eq. 9 was found as well fitting provided that the $F(\Theta)$ relationship is well approximated.

Having the information on the ponding time t_p, the infiltration rate at $t > t_p$ can be computed using a simply modified equation of infiltration with constant-concentration boundary condition, see eqs. 25 and 26.

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